Dynamical scaling of the quantum Hall plateau transition

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Using different experimental techniques we examine the dynamical scaling of the quantum Hall plateau transition in a frequency range $f = 0.1 - 55$ GHz. We present a scheme that allows for a simultaneous scaling analysis of these experiments and all other data in literature. We observe a universal scaling function with an exponent $\kappa = 0.5 \pm 0.1$, yielding a dynamical exponent $z = 0.9 \pm 0.2$.

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Phase transitions between different phases of matter are frequently met in nature, e.g. in the system ice/water, para/ferromagnet, normal/superconductor. The usual classification distinguishes between first and second-order transitions. In a first-order transition the two phases coexist at the transition temperature, in a second-order transition they do not. Such transitions are termed “classical” and occur at non-zero temperature. Different from these types of “classical” phase transitions are quantum phase transitions. Strictly spoken, they occur only at zero temperature [1]. However, as long as the quantum fluctuations governing the transition dominate the thermal fluctuations, we also can observe this quantum phenomenon at $T > 0$.

Second order quantum phase transitions occur at a critical value of a parameter which can be, e.g., the disorder in the metal-insulator transition of two-dimensional electron systems at zero magnetic field or the magnetic field in the transition between Hall plateaus in such systems [2, 3]. The latter one is the target of the investigations presented in this paper.

Generally, when the transition is approached, the correlation length ξ of the quantum fluctuations diverges in form of a power law $\xi \propto |\delta|^{-\gamma}$ with γ being the critical exponent. For a quantum Hall system δ is the distance from a critical energy E_c which can be identified as the center of a disorder-broadened Landau level.

For quantum Hall systems the correlation length corresponds to the localization length $\xi(E)$, which expresses the typical extensions of the wavefunction at energy E and is finite at all energies but E_c:

$$\xi(E) \propto |E - E_c|^{-\gamma} \tag{1}$$

For an infinitely large sample at $T = 0$ K the quantum phase transition from one quantum Hall state at $E < E_c$ to another one at $E > E_c$ happens via a single metallic (extended) state at the critical point E_c; all other states are localized. In contrast, in a finite sample the states with a localization length larger than the sample size L are effectively delocalized, the transition is smoothed onto a finite energy range. Additionally, at non-zero temperature and non-zero measuring frequency further sources of effective delocalization come into play.

The finite size dependence of the wavefunctions was investigated in a number of numerical calculations [4, 5, 6] for non-interacting electrons. They confirmed localization length scaling (Eq. 1) in quantum Hall systems with a universal scaling exponent $\gamma = 2.35 \pm 0.03$, independent of the disorder potential [2]. Short-range interactions are predicted not to change the critical exponent [7]. However, the effect of long-range electron-electron interaction present in the experiments remained unclear.

Experimentally, neither the wavefunction nor the energy E are directly accessible. Instead we measure the conductivity $\sigma_{xx}(B)$ as function of the magnetic field B, observing quasi metallic behavior ($\sigma_{xx} \sim e^2/h$) near some critical field B_c, where the state at the Fermi energy is extended ($\xi(B) > L$), and insulating behavior ($\sigma_{xx} \ll e^2/h$) for localized states ($\xi(B) \ll L$). More generally, the scaling theory predicts the conductivity tensor to follow general functions [8]

$$\sigma_{ij}(B) = G(L/\xi) = G_L(L^{1/\gamma}\delta B) \ . \tag{2}$$

where we have used Eq. 1 and linearized $\delta B = B - B_c \propto E_c - E$ near the critical point. Then the width of the quasi metallic region ΔB, called plateau transition width, follows $\Delta B \propto L^{-1/\gamma}$. When analyzing the ΔB as a function of the sample size L such a prediction was indeed verified experimentally [9], yielding $\gamma = 2.3 \pm 0.2$. Quite recently, this value was also confirmed by indirect measurements of the localization length $\xi(B)$ in the “insulating” variable range hopping regime [10, 11].

Nonzero temperature $T > 0$ or frequency $f > 0$ introduce additional time scales $\tau_T \sim \hbar/k_B T$ or $\tau_f \sim 1/f$ [3]. This time has to be compared to the correlation time $\tau_\xi \propto \xi^z$, which is related to the correlation length ξ by the dynamical exponent z. In a more descriptive approach, the additional time scale τ can be translated into an effective system size $L_{\text{eff}} \propto \tau^{1/z}$. Plugging this into Eq. 2 we find scaling functions

$$\begin{aligned} \sigma_{ij}(f, T{=}0) &= G_f(f^\kappa \delta B) \quad \text{and} \\ \sigma_{ij}(f{=}0, T) &= G_T(T^\kappa \delta B) \end{aligned} \tag{3}$$

with a universal scaling exponent $\kappa = 1/z\gamma$. The plateau transition widths are then given by $\Delta B \propto T^\kappa$ and

$\Delta B \propto f^{\kappa}$. When identifying τ with the phase coherence time, given by excitations with energy $k_B T$ or hf, $L_{\rm eff}$ is interpreted as the phase coherence length and given by the diffusion law $L_{\rm eff}^2 \sim D\tau$. This yields a dynamical exponent $z = 2$ validated in a numerical calculation of the frequency dependence of σ_{xx} for non-interacting electrons [12]. While z is not affected by short-range interactions [13], it has been claimed, that long-range Coulomb interaction changes the dynamical exponent to $z = 1$ [14, 15].

Temperature scaling experiments done so far yield an ambiguous picture. While first experiments on InGaAs quantum wells claimed to observe scaling with a universal exponent $\kappa = 0.42 \pm 0.04$ [16], a number of other experiments reported either scaling with a sample dependent κ [17], or even doubted the validity of scaling at all [18]. For frequency scaling the evidence is even worse. Engel *et al.* [19, 20] claim to observe scaling for two different samples with $\kappa \approx 0.42$, an experiment of Balaban *et al.* [21] contradicts scaling. In addition, all these experiments were not performed in a parameter range where $hf \gg k_B T$ is obeyed and, therefore, any single parameter scaling analysis can not be straightforwardly applied.

To overcome these limitations we combine in this work experiments covering a frequency range $0.1 - 55$ GHz. We have measured the conductivity σ_{xx} in two different experimental setups. In a frequency range from 1 MHz to 6 GHz [10, 22] we use coaxial conductors, replaced by wave-guides for higher frequencies from 26 GHz to 55 GHz [23, 24]. Our experiments are complemented with data from literature [19, 20, 21, 25] and present a two-variable rescaling scheme for frequency and temperature scaling. Using all these data [10, 19, 20, 21, 22, 23, 24, 25] we will demonstrate a universal function with a universal scaling exponent $\kappa = 1/z\gamma = 0.5 \pm 0.1$, yielding a dynamical scaling exponent $z = 0.9 \pm 0.2$.

Frequencies up to 6 GHz are realized in a coaxial reflections setup as described in detail in Refs. [10, 22]. The sample, patterned into Corbino geometry, acts as load of a coaxial cable fitted into a ^{3}He/^{4}He dilution refrigerator. The sample conductivity is derived from reflection measurements. The 2DES used in these experiments was realized in an AlGaAs/GaAs heterostructure with electron density $n_e = 3.3 \cdot 10^{15}$ m^{-2} and mobility $\mu_e = 35$ m^2/Vs. Traces of $\sigma_{xx}(B)$ shown in Fig. 1a reveal a peak at every transition between quantum Hall states. The spin-split transitions are resolved up to a filling factor $\nu = n_e h/eB = 6$. The transition widths ΔB measured as the full width at half maximum of the peaks are shown in Fig. 1c [29]. For $f \leq 1$ GHz ΔB is governed by the temperature $T \approx 0.1$ K of the 2DES as deduced from temperature dependent measurements at $f = 0.2$ GHz. Above 2 GHz frequency scaling $\Delta B \propto f^{\kappa}$ takes over.

In a second experiment we use waveguides to access frequencies from 26 to 55 GHz. The sample with the 2DES,

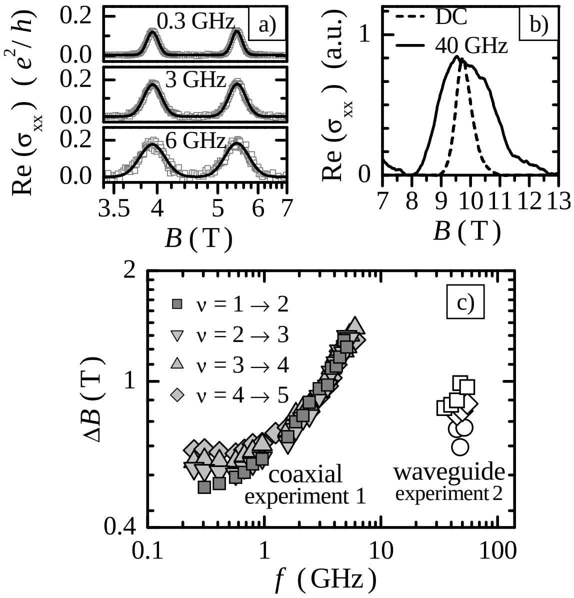

FIG. 1: (a) Conductivity of sample 1 measured in the coaxial setup for different frequencies. Here shown are the conductivity peaks at the $\nu = 4 \rightarrow 3$ and $\nu = 3 \rightarrow 2$ plateau transition. (b) Conductivity of sample 2 at the $\nu = 2 \rightarrow 1$ transition, measured in the waveguide reflection setup for very high frequencies and in a classical Hall setup for DC.
(c) Frequency dependence of the width of the quantum Hall plateau transitions, measured for $\nu > 2$ as full width and for $\nu = 2 \rightarrow 1$ as half width at half maximum [23, 29]. For the waveguide experiment different symbols denote different samples from the same waver resp. different cooldown cycles with slightly different carrier densities.

realized in a standard AlGaAs/GaAs heterojunction, is placed at the end of the waveguide and partially reflects the incident microwave. We modulate the carrier density using a thin front gate to discriminate the 2DES contributions from other reflections, for details see [23, 24]. The electron density and mobility are $n_e = 3.6 \cdot 10^{15}$ m^{-2} and $\mu_e = 10$ m^2/Vs. The use of a low mobility sample ensures that hf (0.2 meV at 50 GHz) is smaller than the Landau Level width (~ 1 meV). For higher Landau levels we do not observe a distinct spin splitting and thus concentrate for this experiment on the $\nu = 1 \rightarrow 2$ transition. The measured conductivity $\sigma_{xx}(B)$ at a temperature of $T = 0.3$ K is shown in Fig. 1b, the evaluated transition widths [23, 29] are depicted in Fig. 1c.

The combination of our two experimental techniques allows to investigate the scaling behavior of the quantum Hall plateau transition in a large frequency range. Additionally, we can compare our results to all other frequency scaling experiments. Table I summarizes the wide ranges of the parameters like frequency, mobility, density, temperature, filling factor, and material, which were covered by the different experiments [10, 19, 20, 21, 22, 23, 24, 25]. The observed frequency dependencies of the Hall

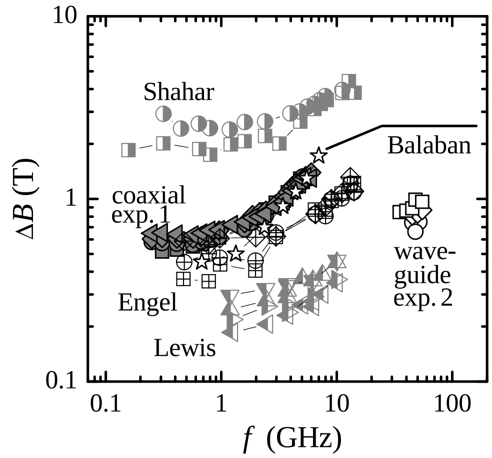

FIG. 2: Comparison of the transition width vs. frequency for all experiments [10, 19, 20, 21, 22, 23, 24, 25]. Multiple symbols for the same experiment denote different samples or cooldowns (waveguide), transitions (coaxial, Lewis) or temperatures (Engel, Lewis, Shahar) as presented in Table I.

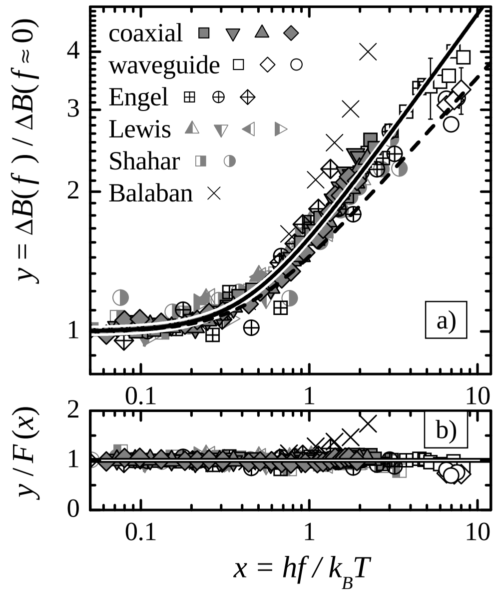

FIG. 3: (a) Normalized plateau transition width $y = \Delta B(f,T)/\Delta B(f \approx 0, T)$ vs. dimensionless parameter $x = hf/k_BT$ for all data presented in Table I and Fig. 2. The solid line results from a fit of $F(x) = (1 + (\alpha x)^2)^{\kappa/2}$ (Eq. 5) to all data except those for highest frequencies $f > 20$ GHz and the Balaban experiment ($\alpha \approx 2$, $\kappa = 0.5 \pm 0.1$). The dashed line depicts a reduced $\kappa = 0.4$.
(b) Rescaling of $y = \Delta\nu(f,T)/\Delta\nu(f \approx 0, T)$ by the scaling function $F(x)$ allows to judge the quality of scaling.

plateau transition width ΔB are summarized in Fig. 2.

Since most of these data do not fulfill $hf \gg k_BT$, we have to take into account the influence of both frequency f and temperature T for our analysis. Therefore, the single-parameter scaling functions for the plateau transition (Eq. 3) are modified using a two-variable scaling analysis [3, p.327]

$$\sigma_{ij}(T, f) = G_{T,f}(T^\kappa\, \delta B, f^\kappa\, \delta B) \ . \tag{4}$$

Both hf and k_BT set an energy scale. Since frequency and temperature act as independent processes it is a reasonable ansatz to sum the energies squared [30], resulting into a combined energy scale $\Gamma = [(\alpha hf)^2 + (k_BT)^2]^{1/2}$. The factor α is of the order of unity and covers the differences of the effects of frequency and temperature. Using this simple model the transition width scales as $\Delta B \propto \Gamma^\kappa$ and can be rewritten as

$$\Delta B_s(T, f) = \Delta B_s(T) \left(\sqrt{1 + \left(\frac{\alpha\, hf}{k_BT}\right)^2} \right)^\kappa \tag{5}$$

with a prefactor $\Delta B_s(T)$ only depending on temperature and on the individual sample s.

We use this equation to combine all data presented in Fig. 2 in a single graph as shown in Fig. 3a. All the transition widths $\Delta B(T, f)$ measured in the different experiments are normalized to the DC width $\Delta B(T, f \approx 0)$ and plotted versus the ratio $x = hf/k_BT$ of frequency and temperature [26]. The lowest temperature T of the 2DES is estimated in the following way: Most authors state the exponent for the temperature dependence of $\Delta B(T)$. Combined with the low frequency width ΔB at high temperatures where the 2DES still couples thermally to the liquid ^{3}He/^{4}He bath we extract the lowest 2DES-temperature from the $f \to 0$ saturation width.

All data except those from Balaban *et al.* [21] fall on top of our data and each other, independent of material, mobility, density, experimental technique, temperature and filling factor, including the quantum Hall to insulator transition analyzed in Ref. [20]. The single deviation observed in Ref. [21], accompanied by deviations from temperature scaling, is probably caused by macroscopic

TABLE I: Key data of the compared experiments: 2DES mobility μ_e in m^2/Vs, analysed range of filling factor ν, 2DES temperature T, and frequencies f. The 2DES used in the experiment of Shahar *et al.* resides in InGaAs, for the other Experiments AlGaAs/GaAs heterostructures were used.

experiment	$\mu_e \left(\frac{\mathrm{m}^2}{\mathrm{Vs}}\right)$	ν	T (K)	f (GHz)
coaxial, exp.1 [10, 22]	35	1-5	0.1	0.1-6
waveg., exp.2 [23, 24]	10	1-2	0.3	35-55
Engel *et al.* [19]	4	1-2	0.14-0.5	0.2-14
Shahar *et al.* [20]	3	0-1	0.2-0.43	0.2-14
Balaban *et al.* [21]	3	1-2	0.15?	0.7-7
Lewis [25]	50	3-5	0.24-0.5	1-10

inhomogeneities, which was shown to spoil any universal scaling behavior [27]. We therefore exclude this data from the following scaling analysis.

The observation of a universal function $F(x)$ in Fig. 3 clearly reveals the universality of the quantum phase transition between different Hall plateaus and into the Hall insulator. We can now determine the associated universal scaling exponent κ by a fit of Eq. 5, omitting only the highest frequencies $f > 15$ GHz from the waveguide experiment, which will be treated separately. The fit, shown as solid line in Fig. 3, yields $\alpha \approx 2$ for the crossover parameter between frequency and temperature and a scaling exponent $\kappa = 0.5 \pm 0.1$.

Within their individual 2σ error the data from the second experiment using the waveguide technique also agree with the fit function $F(x)$, but Fig. 3 also reveals that their combined statistical weight indicates a slightly lower scaling exponent κ between 0.5 and 0.4. This reduction of κ towards its lower limit $\kappa = 0.2$ for a non-interacting 2DES [2, 13] possibly hints to a partial screening of Coulomb interaction of the electrons, caused by the gate on top of the sample.

Now that we have determined the universal scaling exponent $\kappa = 1/z\gamma = 0.5 \pm 0.1$ of the frequency and temperature scaling, we would like to separate the dynamical exponent z and the critical exponent γ. Recent experiments exploited the frequency [10] and the temperature dependence [11] of the conductivity in the variable range hopping regime, which allowed to determine the localization length $\xi(B)$. Both observed a scaling behavior $\xi \propto |\delta\nu|^{-\gamma}$ with a universal critical exponent $\gamma = 2.3 \pm 0.2$ in agreement with earlier size scaling experiments [9] and astoundingly with the value obtained in numerical studies for non-interacting electrons [4, 5, 6]. With this universal scaling exponent $\gamma \approx 2.3$ we can then deduce a dynamical exponent $z = 1/\gamma\kappa = 0.9 \pm 0.2$ in agreement with theoretical predictions [14, 15].

In conclusion, we have developed a method to combine data from all experiments on frequency scaling of the quantum Hall plateau transition. We find a universal scaling function independent of material, density, mobility, experimental technique, temperature and filling factor, which clearly demonstrates the universal nature of this quantum phase transition. We determine the universal scaling exponent of an interacting quantum Hall system to $\kappa = 0.5 \pm 0.1$ and, using the critical exponent $\gamma \approx 2.3$, the dynamical exponent to $z = 0.9 \pm 0.2$.

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[29] For the $\nu = 1 \rightarrow 2$ transition in the lowest Landau band only the half width at the high filling side was used to determine ΔB, the critical point B_c was kept fixed at its low-frequency value. This is necessary because an additional shoulder appears in σ_{xx} which is assigned to the presence of attractive impurities in the sample [28]. Therefore, a definition of ΔB by the full width is no more possible.
[30] Simply adding the energies ($\Gamma = k_BT + \alpha hf$) does not fit our data.